Comments on the theory and results of simulations of Electron Cyclotron Drift Instability

A. Smolyakov[1]
D. Sydorenko[1,2], S. Janhunen[1,3], T. Zintel[1], M. Sengupta[1], O. Chapurin[1],
Y. Raitses[4], I. Kaganovich[4],
A. Powis[4], J. Carlsson[4], J.P. Boeuf[5], F. Taccogna[6]

[1]University of Saskatchewan
[2]University of Alberta
[3]U Texas at Austin
[4]Princeton University and PPPL
[5]LAPLACE & Université Paul Sabatier
[6]CNR, Univ Bari

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Outline/Summary

- Comments on the linear and nonlinear transition to ion-sound, particle stochastization/nonlinear broadening mechanisms
- Previously (published, PoP Janhunen 2018a,2018b): 1D, 2D (theta-r) simulations
- Parametric studies of 1D ECDI (Test case 1a)
- 2D Full cylinder (theta-r) simulations
- 2D (theta-z) simulations, Test case 2a- Wide box simulations

In Conclusions

- 1D nonlinear simulations are generally similar among several groups (Laplace, Bari, Usask)
  - –Cnoidal waves in ion density, benign electron density, $k_{y0}V_{E0} = \omega_{ce}$ cyclotron resonance modes, condensation toward longer wavelengths, the mode frequency around ω_{pi} , axial current depends on the reinjection parameters/length, increases transport, ion trapping signatures, somewhat sensitive to simulation parameters (NPP), puzzling dependence of anomalous current (linear with E, but strong dependence on the magnetic field? Further studies are warranted but it is not clear if all relevant physics is included in such 1D.
- 2D (theta-radial) in many aspects (cnoidal waves, ω_{pi} harmonics) are similar to 1D, but radial modes included-> slow but intense MTSI modes, parallel heating, and structures in axial current.
- 2D theta-z simulations show common features: 2mm wave, higher axial temperature (Laplace, LLP, Usask). Wide box simulations (Usask) show high frequency modes, electron streamers current, and long wavelength modes/vortex. High frequency modes in HT (Litvak et al, Lazurenko et al)?
- From the theory perspective, the cyclotron resonances are not sufficiently broadened in 1D simulations, nor in 2D (theta-radial). It is somewhat unexpected that finite kz modes (along the magnetic field) do not make the mode ion-sound like via the linear mechanism (kz is not large enough, sheath effect) . Stochastic diffusion regimes in 2D theta-z and quasilinear regimes applicability are not clear

Electron cyclotron drift instability driven by ExB drift

- Robust instability demonstrated in 1D and 2D PIC simulations by many groups; easy to observe in a relatively simple PIC simulation (from 70s)
- Very effective electron heating mechanism
- Also axial anomalous current

What do we see in simulations? Can we explain and predict it?

Ion sound like turbulence and transport? Can we use quasilinear type formula for unmagnetized plasma which worked in many cases, e.g.?

$$\frac{\nu^*}{\omega_{pe}} \sim C \frac{v_d}{v_{Te}} \frac{T_e}{T_i} \left(\frac{m_e}{m_i} \right)^{1/4}$$

"… for weakly magnetized plasmas, $\omega_{ce} < \omega_{pe}$, the anomalous plasma resistivity for the electric current transverse to the magnetic field is similar…
Sagdeev,Reviews of Plasma Physics 1973, v7

Active discussion whether the instability with magnetic field
is similar to the ion sound instability for zero magnetic field B=0 was ongoing since 70s

What do the simulations say?

Lampe at al - looks like ion sound;
Forslund et al -very different from B=0 case
Lindman (1985), previously unpublished extensive studies
at LANL and comparison with Biskamp et – finite B and B=0 are different

Does the turbulence driven by ExB flow looks like ion sound?
Is the quasilinear weak turbulence theory applicable?

Katz et al., 2015, Lafleur et al 2016- Unmagnetized quasilinear theory to describe anomalous electron transport; wave kinetic equation for ion sound waves…
Tsikata et al –Ion sound signatures in experimental spectra

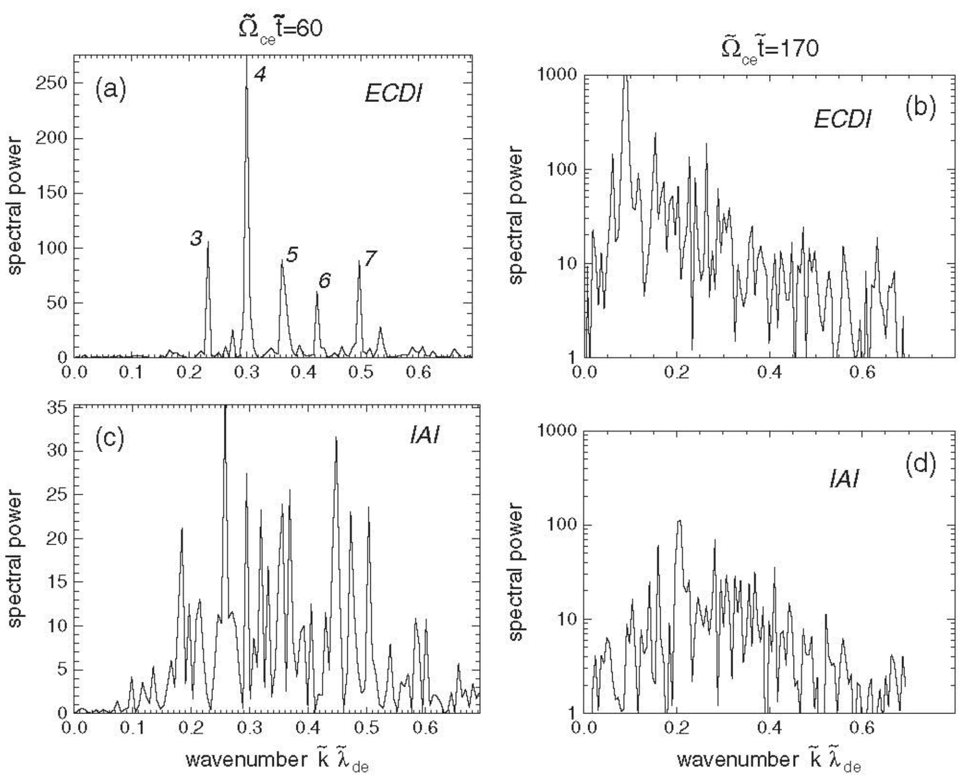
Why don’t we compare two cases $B = 0$ and $B \neq 0$ directly in simulations?

Our initial 1D simulations have shown large differences, not pursued …
Simulate longer time?

Direct comparison Muschietti and Lembege, JGR 2013

Identical parameters except B: ECDI shows much stronger spectral power

MUSCHIETTI AND LEMBÈGE: ELECTRON CYCLOTRON DRIFT INSTABILITY



ECDI

IAI, B=0

Figure 13. Snapshots of the electrostatic energy spectrum at two times, $\tilde{\Omega}_{ce}\tilde{t} = 60$ and 170, comparing two different simulation runs: (a, b) ECDI reference run with $\tilde{B}_0 = 5$ and (c, d) IAI run with $\tilde{B}_0 = 0$. Apart from $\tilde{B}_0$, other parameters are identical (cf. Table 2).

Ion sound growth rate peaks up at a finite angle to the electron flow angle; respectively jet spectra of ion sound turbulence; Kadomtsev, Silin, Urypin..

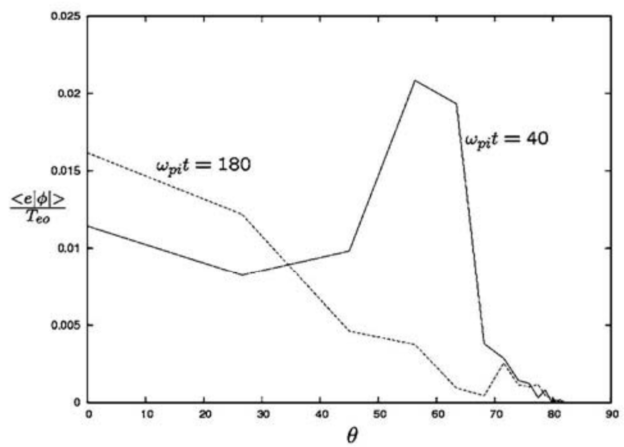


Figure 2. Electrostatic fluctuation amplitude versus mode angle with respect to the electron drift at time levels corresponding to the linear phase (solid curve) and the saturation phase (dashed curve). Averaging in time is made over 10 time steps.

Sydora, Collisonless simulations of the ion sound, 2006

How can we have the ion sound with magnetized electrons?

Two types of sound waves in magnetized plasmas

- Standard sound requires some propagation along the magnetic field, for

$$\omega < k_z v_{Te} \quad n_e = \frac{e\phi}{T_e} n_0$$

$$\omega^2 = k^2 c_s^2 / (1 + k^2 \lambda_D^2)$$

  with unmagnetized ions one gets

$$k_z = 0$$

- Short wavelength sound, for strictly perpendicular propagation,

$$k_\perp^2 \rho_e^2 > 1 \qquad n_e = \frac{e\phi}{T_e} n_0 \left[1 - \exp\left(-k_\perp^2 \rho_e^2\right) I_0(k_\perp^2 \rho_e^2)\right] \to \frac{e\phi}{T_e} n_0$$

$$\exp\left(-k_\perp^2 \rho_e^2\right) I_0(k_\perp^2 \rho_e^2) \to \frac{1}{k_\perp \rho_e} \to 0$$

- Short wavelength sound is the thermal extension of the lower hybrid modes

$$\omega^2 = \omega_{LH}^2 (1 + k_\perp^2 \rho_e^2) \to \omega_{LH}^2 k_\perp^2 \rho_e^2 = k_\perp^2 c_s^2 \qquad \omega_{LH}^2 = \omega_{ce} \omega_{ci}$$

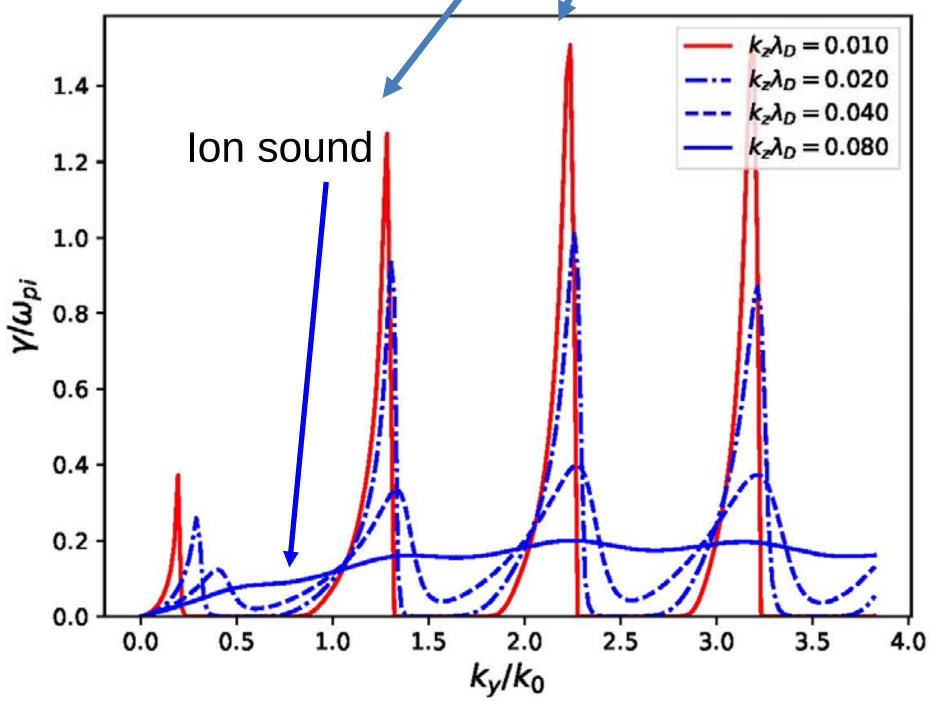
Electron cyclotron drift instability: Reactive (fluid) resonances regimes and transition to ion sound

$\omega - k_y v_{ExB} = m\omega_{ce}$

$\omega \ll k_y v_{ExB} \sim m\omega_{ce}$

Thermal spreading
due to $k_z v_{te}$

for relatively large k_z transition
to unmagnetized ion sound
instability driven by v_{ExB} beam
(Gary, Sanderson, Lampe ,
Cavalier)



In the context of ExB discharges
Adam et 2004, Boeuf, Lafleur, Ducrocq,..

Why and how the ECDI would become the ion sound, or something similar?

Basic (and most strong) ECDI has a reactive (fluid type) resonance instability

Resonances may be broadened by

a) collisions, real and numerical;

b) finite wavevector along the magnetic field;

c) nonlinear effects/broadening

Broadened instability(s) looks similar to the ion sound/lower hybrid for longer wavelengths?

Kinetic Electron Cyclotron Drift Instability (ECDI): Bernstein modes destabilized by ExB drift

in the direction of the azimuthal drift, electrostatic, unmagnetized ions, no gradients, collisions, etc

Linear dispersion relation

$$\epsilon(\omega, \mathbf{k}) = 1 + \mu_i(\omega, \mathbf{k}) + \mu_e(\omega, \mathbf{k}) = 0,$$

$$\mu_i = -\frac{\omega_{pi}^2}{\omega^2}$$

$$\mu_e = \frac{1}{k^2\lambda_D^2}\left[1 + \frac{\omega - \mathbf{k}\cdot\mathbf{v}_0}{\sqrt{2}k_z v_e}\sum_{m=-\infty}^{\infty}\exp(-b)\,\mathrm{I}_m(b)\,\mathrm{Z}\left(\frac{\omega - \mathbf{k}\cdot\mathbf{v}_0 + m\omega_{ce}}{\sqrt{2}k_z v_e}\right)\right],$$

where $b = k_\perp^2\rho_e^2$, $k \equiv \mathbf{k}$, $\lambda_{De} = \frac{\epsilon_0 T_e}{n_e q_e^2}$, $v_{e,i} = T_{e,i}/m_{e,i}$, $k_i^2 = c_s^2/\lambda_{De}^2$, $Z(\xi)$ is the plasma dispersion function, $I_m(x)$ is the modified Bessel function

In the cold plasma limit, magnetized Buneman instability

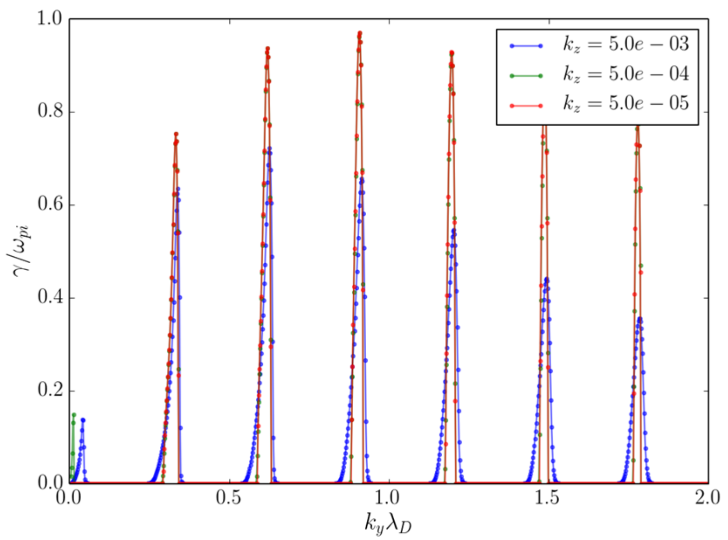
$$1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(\omega - kv_0)^2 - \Omega_{ce}^2} = 0$$

Buneman 1962, Aref'ev, Gordeev, Rudakov 1969..., Lampe.., Forslund .., Gary .. Stepanov,, Lominadze, 1970s

Reactive (fluid) cyclotron resonances

$$k_z = 0$$

$$\mu_e = \frac{1}{k^2\lambda_{De}^2}\left[1 - \exp\left(-k^2\rho_e^2\right) I_0\left(k_\perp^2\rho_e^2\right) - 2\left(\omega - k_x v_E\right)^2 \sum_{m=1}^{\infty} \frac{\exp\left(-k_\perp^2\rho_e^2\right) I_m\left(k_\perp^2\rho_e^2\right)}{\left(\omega - k_x v_E\right)^2 - m^2\Omega_{ce}^2}\right]$$

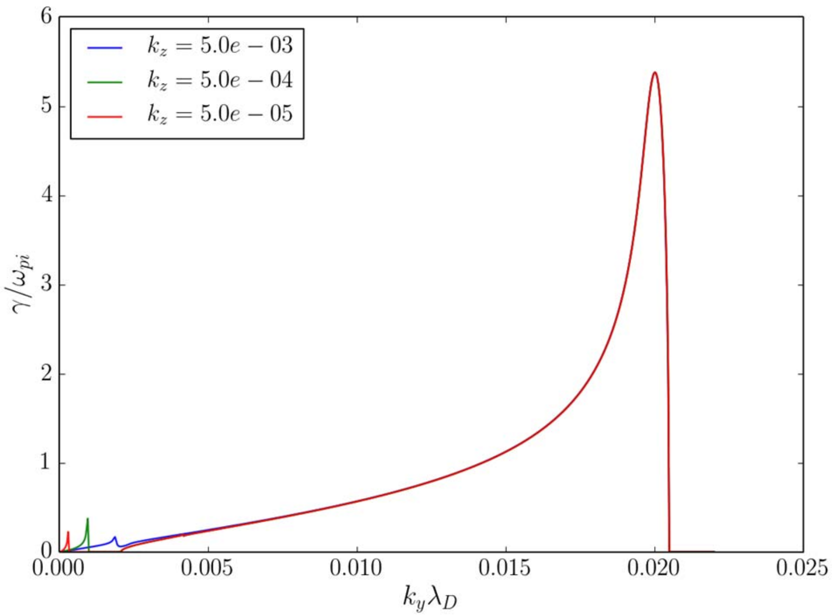


For ion sound

$$\mu_e = \frac{1}{k^2\lambda_{De}^2}$$

The cold plasma limit, magnetized Buneman instability

$$1-\frac{\omega_{pi}^2}{\omega^2}-\frac{\omega_{pe}^2}{(\omega-kv_0)^2-\Omega_{ce}^2}=0$$



What is cold?

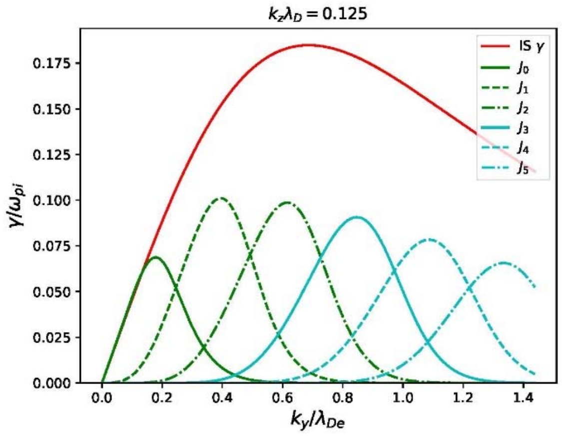
Long wavelength limit

$$k_\perp{}^2\rho_e{}^2 << 1$$

Fluid modeling is possible?

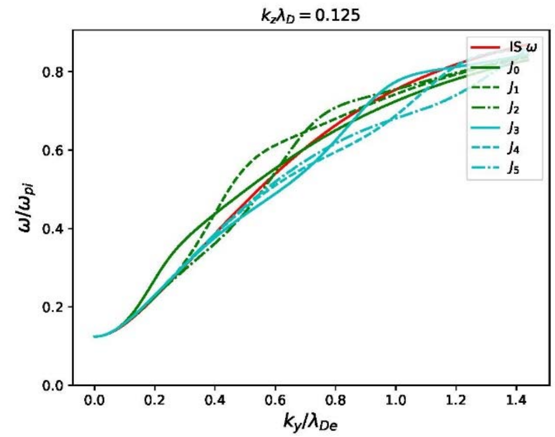
Structure of cyclotron resonances and transition to ion sound modes

Thermal spreading due to $k_z \mathrm{v}_{\mathrm{te}}$



for relatively large k_z transition to unmagnetized ion sound instability driven by v_{ExB} beam (Gary, Sanderson, Lampe , Cavalier)

Linear mechanism



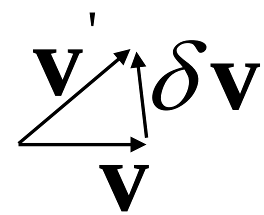
Each harmonic looks like a sound wave
Unmagnetized sound wave is a sum of many

Nonlinear resonance broadening

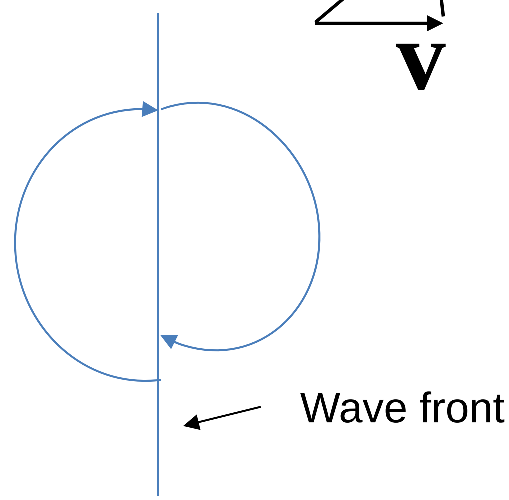
Formally

$$K_e^{nl} = \frac{1}{k^2\lambda_{De}^2}\left[1 + \left(\frac{\pi}{2}\right)^{1/2}\frac{(\omega - kv_E)}{kv_e} \times \times \cot\left(\pi\frac{\omega - kv_E + ik^2D_{nl}}{\Omega_{ce}}\right)\right].$$

$k^2D_{nl} > \Omega_{ce}$, which is equivalent to the condition $(D_{nl}\tau_c)^{1/2} > \lambda/2)$, $\cot\left(ik^2D_{nl}/\Omega_{ce}\right) \simeq -i$ gives the Landau type (ion sound)



$$\delta r = \frac{\delta v}{\omega_{ce}}$$



Phase decorrelation for $\delta r > \lambda / 2$

$$\frac{\tilde{v}_E}{v_e} > 1 \qquad or \; \frac{\tilde{v}_E}{v_e} > \frac{1}{\sqrt{k\rho_e}}$$

$$v_e = \max(v_{Te}, V_{E0})$$

Generally is not satisfied in our 1D simulations
And seems to be marginal (threshold) for 2D (theta-z)

Particle stochastization, resonance broadening and quasilinear theory for a single (coherent) wave

- Generally quasilinear theory requires a wide spectrum with overlapping resonances
- Our (and others) simulations show the coherent wave (+ some longer wavelength)

In general, particle stochastization is possible for a single coherent wave

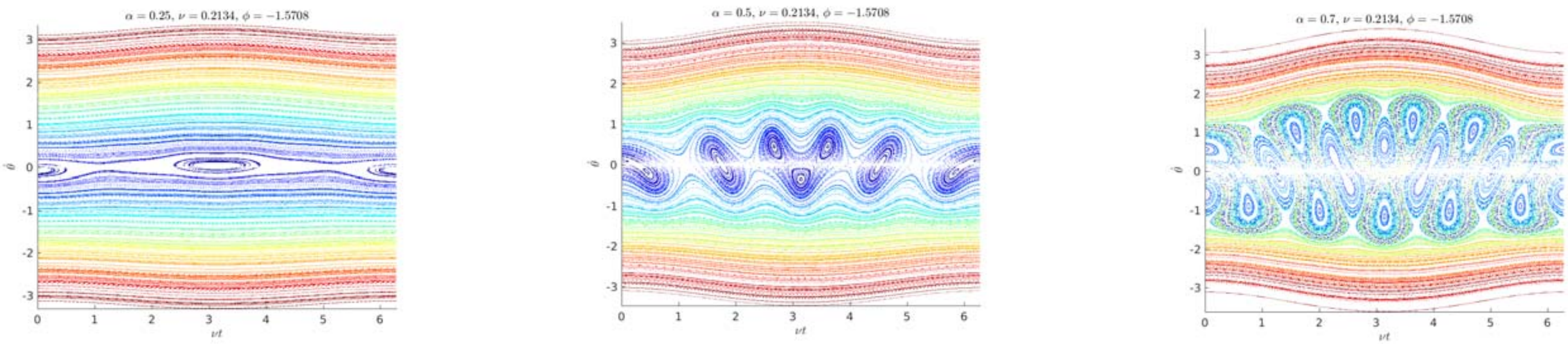
$$\frac{d\theta^2}{dt^2} + \omega_{ce}{}^2\theta = \alpha \sin\left(ky - (\omega - k_y v_0)t\right)$$

$$\frac{d\theta^2}{dt^2} + \omega_{ce}{}^2\theta = \alpha_0 \sum J_l\left(k_\perp \rho\right)\sin\left((\omega - k_y v_0 - l\omega_{ce})t\right)$$

l=0,1,2, resonances; should overlap for stochastisation and diffusion (quasilinear theory?)
Hamiltonian system with 2 degrees of freedom, KAM theorem (stable trajectories exist)
Stochastization is never complete (diffusion/heating is limited)

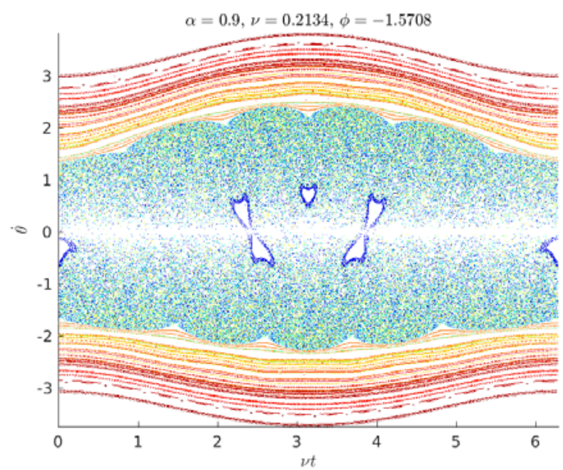
Chirikov, Zaslavsky, Karney, Smith, Kaufman, …many others

Resonances and particle stochastization



resonances of different order

Stochastic region for low electron energies, S. Janhunen, …

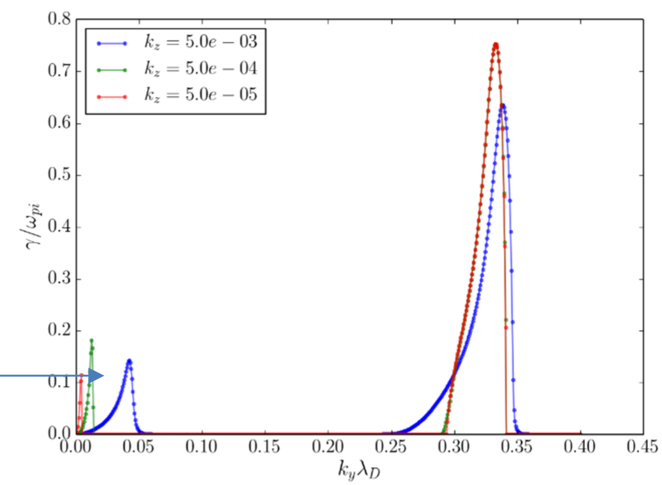


It is not clear if stochastisation occurs in the simulations;
1D and 2D cases are different

Gradient density/magnetic field effects - Lower hybrid instability

Long wavelength mode –typically more difficult to stabilize

Long wavelength mode are found in our 2D wide box simulations



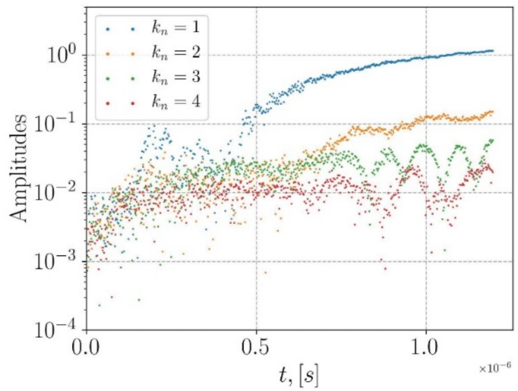
Theoretical linear growth rate with density gradient included

Some conclusions from most recent work

- Robust instability demonstrated in 1D and 2D PIC simulations by many groups
- Very effective electron heating mechanism (often difficult to control in PIC due to CFL condition; requires small time steps)
- 1D simulations show cyclotron modes $\omega << k_y v_0 = n\omega_{ce}$
- Ion density fluctuations typically have larger amplitude than electrons $\tilde{n}_i > \tilde{n}_e$ generally requires finite wavelength compared to the Debye $k^2 \lambda_{De}{}^2 \geq 1$ Ions show wave steepening and breaking at small scales, cnoidal wave structure, also evident in time spectra as ω_{pi} harmonics
- Nonlinear development of large scale modes (absent in linear theory), via modulational instability mechanism, requires azimuthally wide simulation box

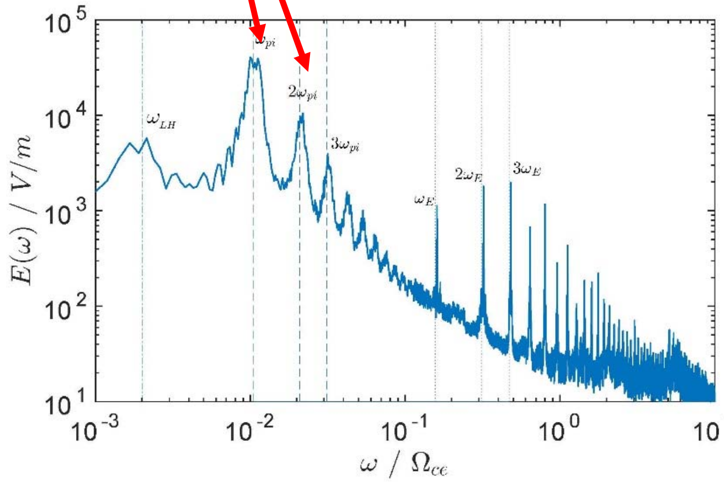
Nonlinear (1D) simulations of electron cyclotron drift instability

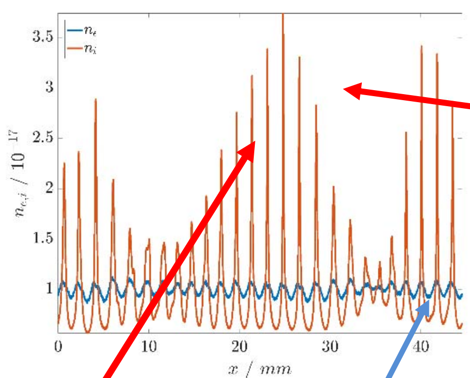
Janhunen et al (paper 1,1D), Phys Plasmas 2018
Janhunen et al (paper 2,2D, r-theta), Phys Plasmas 2018



Instability starts from the most unstable mode, proceeds nonlinearly at the fundamental resonance

ω_{pi} harmonics



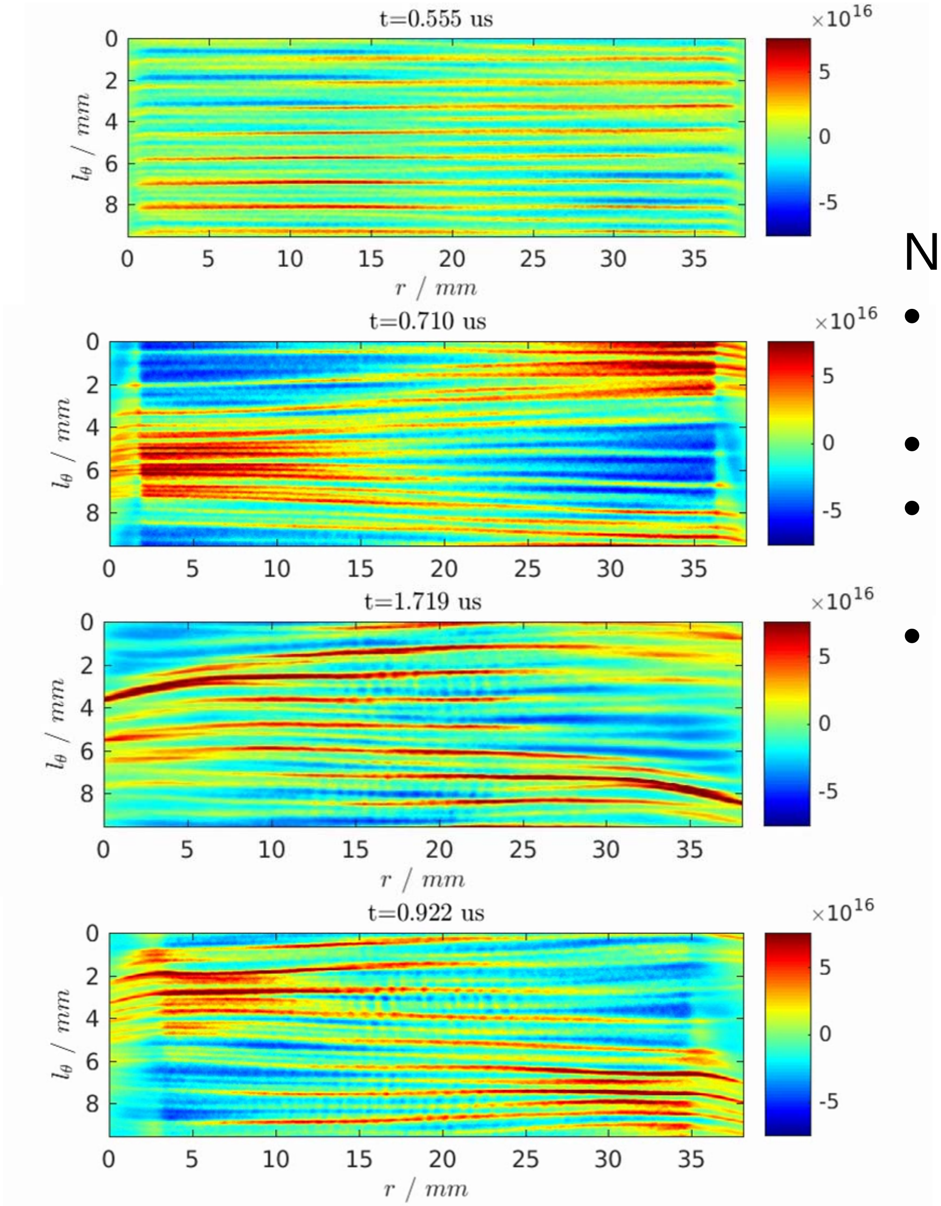


Nonlinear development of the long wavelength envelope: inverse energy cascade

ions

Electron density

Electron cyclotron drift instability in 2D (r-theta) simulations (Janhunen PoP 2018)



Similar to 1D features of cyclotron resonance modes, ion cnoidal waves, long wavelength modulations are observed in 2D r-theta simulations

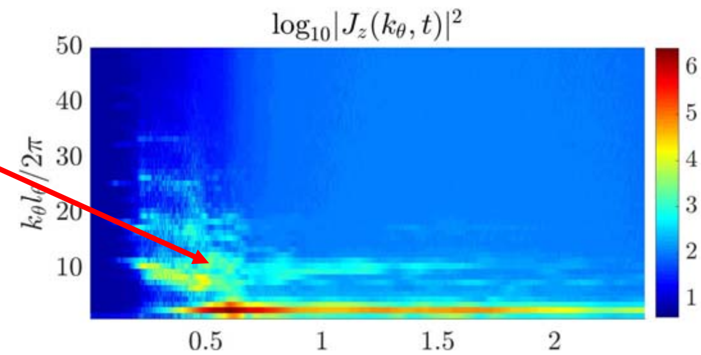
New 2D features due to :

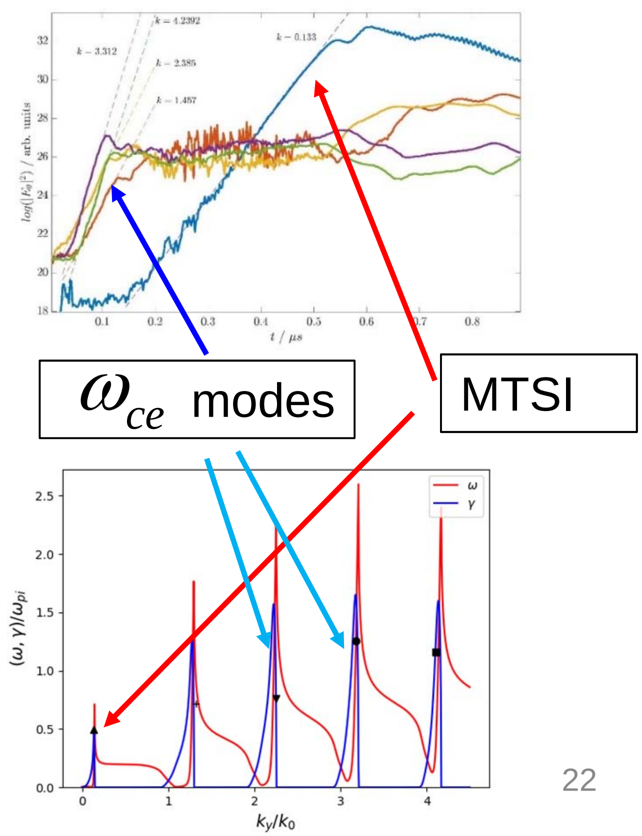
- Radial (along B) structure due to Modified Two Stream Instability
- Strong parallel heating
- Axial current is dominated by the long wavelength mode
- Radial structure in axial anomalous current

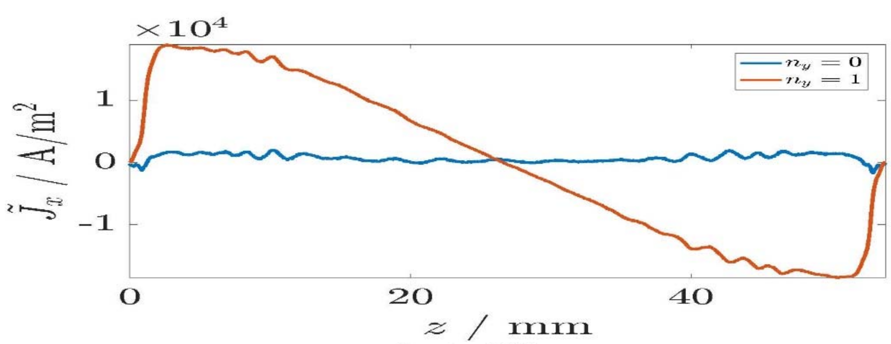
ECDI and MTSI instabilities in 2D (r-theta) (Janhunen PoP 2018)

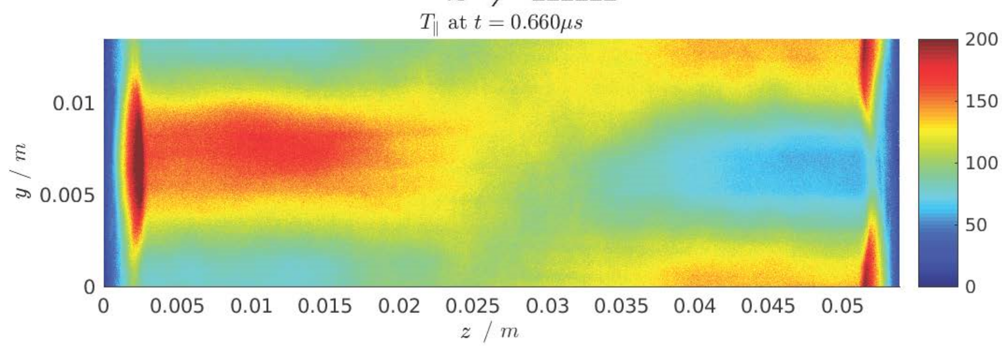
New features in 2D simulations

- Axial current is dominated by the long wavelength mode
- Radial (along B) structure due to Modified Two Stream Instability $\lambda / 2$
- Strong parallel heating
- Strong radial structure in axial anomalous current and heating (n=1 azimuthal)



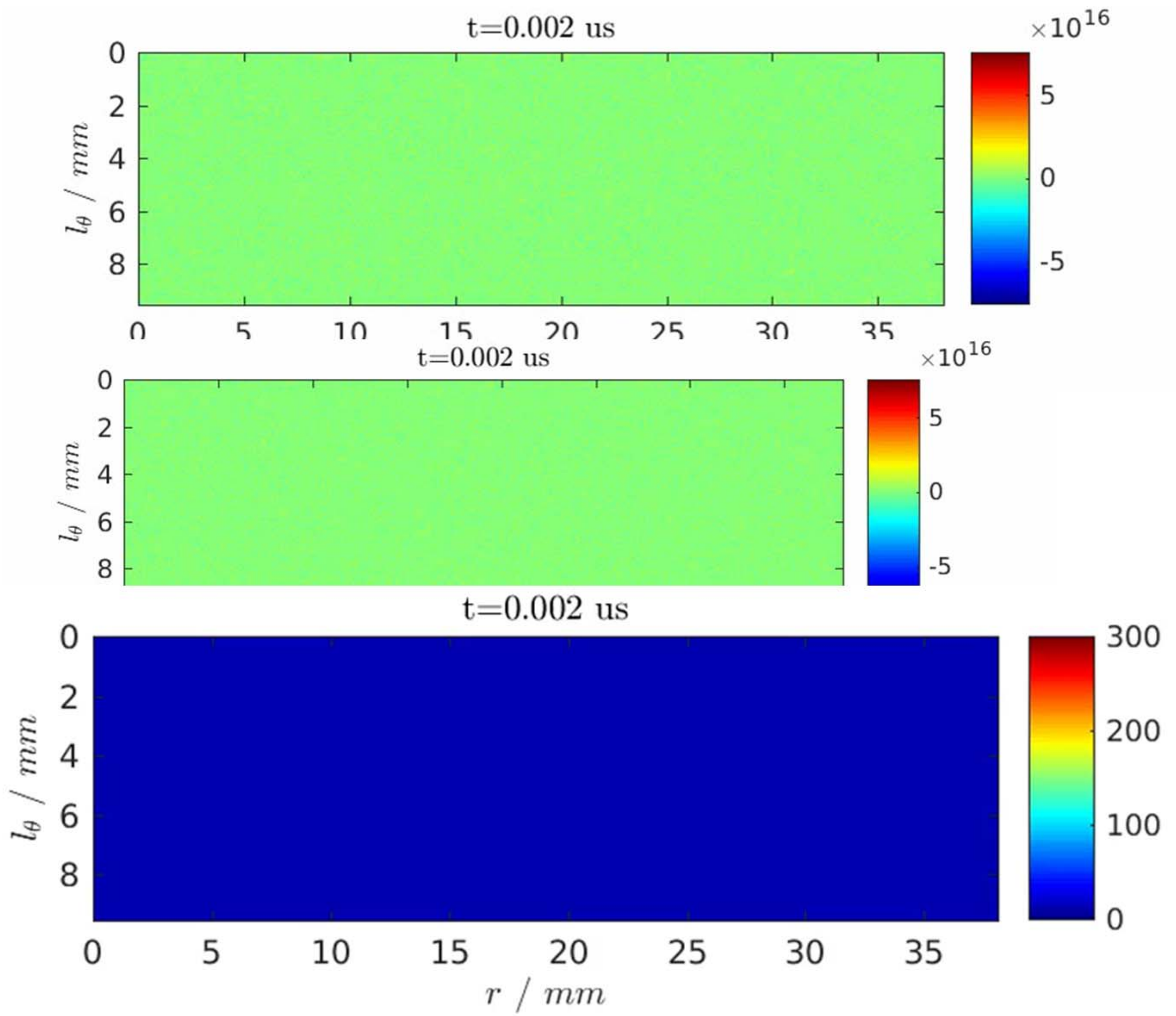






Evolution of the electron cyclotron drift instability in two-dimensions
Janhunen et al,
Physics of Plasmas **25**, 082308 (2018); https://doi.org/10.1063/1.5033896

Movies are at PoP website as supplemental data



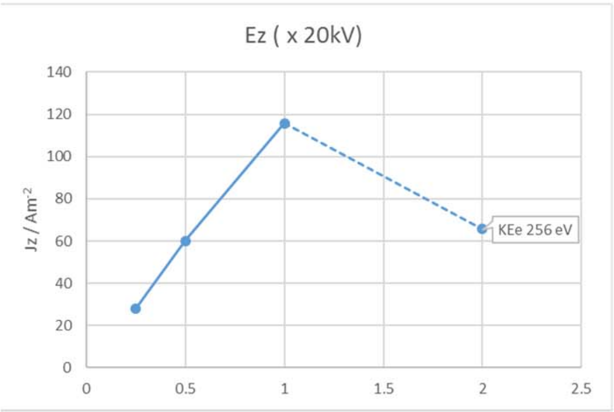
Parametric studies of 1D ECDI

T Zintel, U Sask

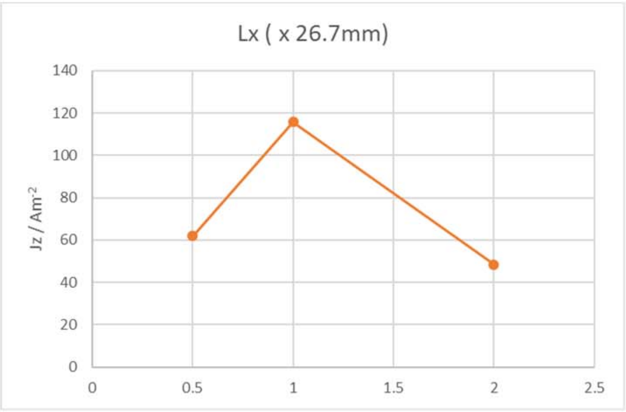
Test case 1a, as per parameters by Taccogna,
but lower density, 4x10^16 cm^-3

Some conclusion

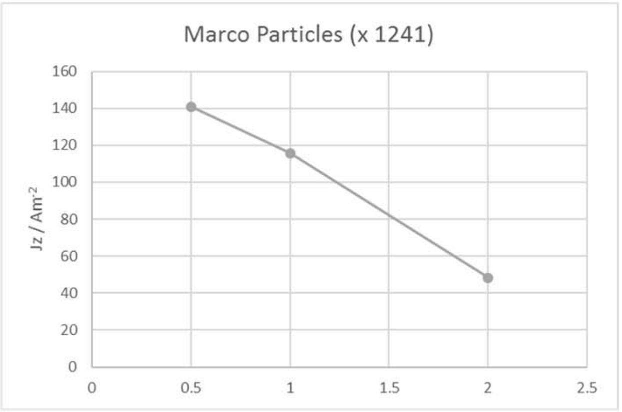
- Reducton of transport with NPP
- Linear scaling with E
- Oscillations in long runs, 70 mks?
- ….



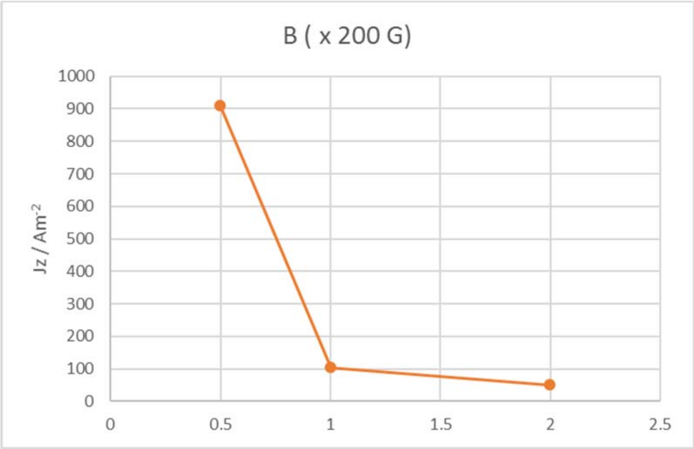
Electric field



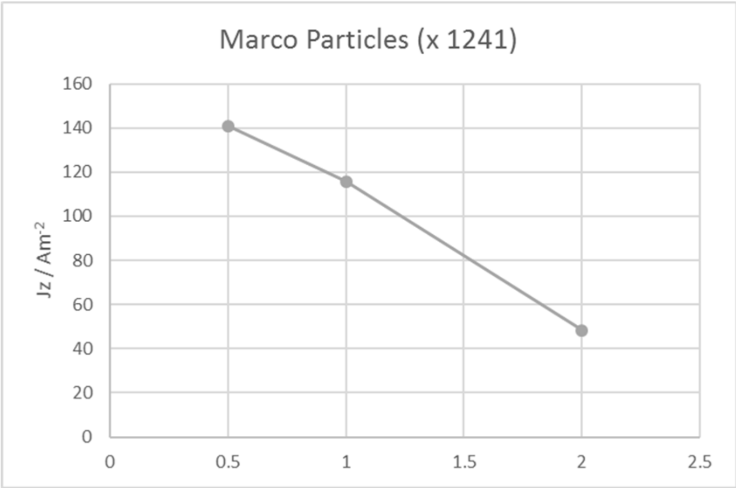
Length



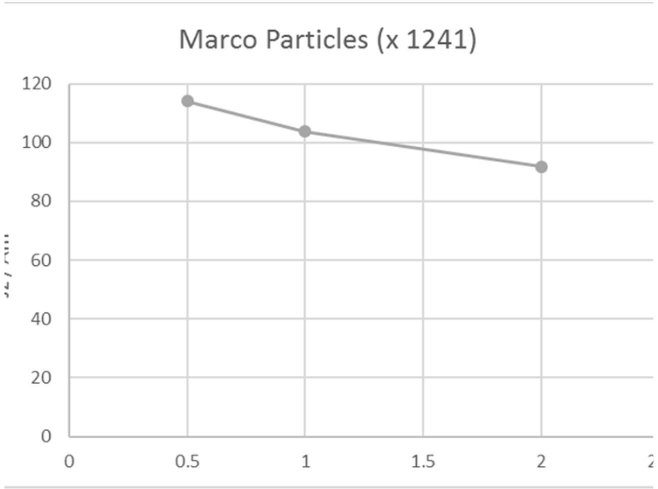
NPP



Reduced magnetic field



Short simulation, 10 mks



Long simulation, 70 mks

Full cylinder 2D (simulations)

M Sengupta, U Sask

ne=ni=1e17 m^-3

Ri = 0.035 m , Ro = 0.05 m

B0 at Ri = 0.015 T

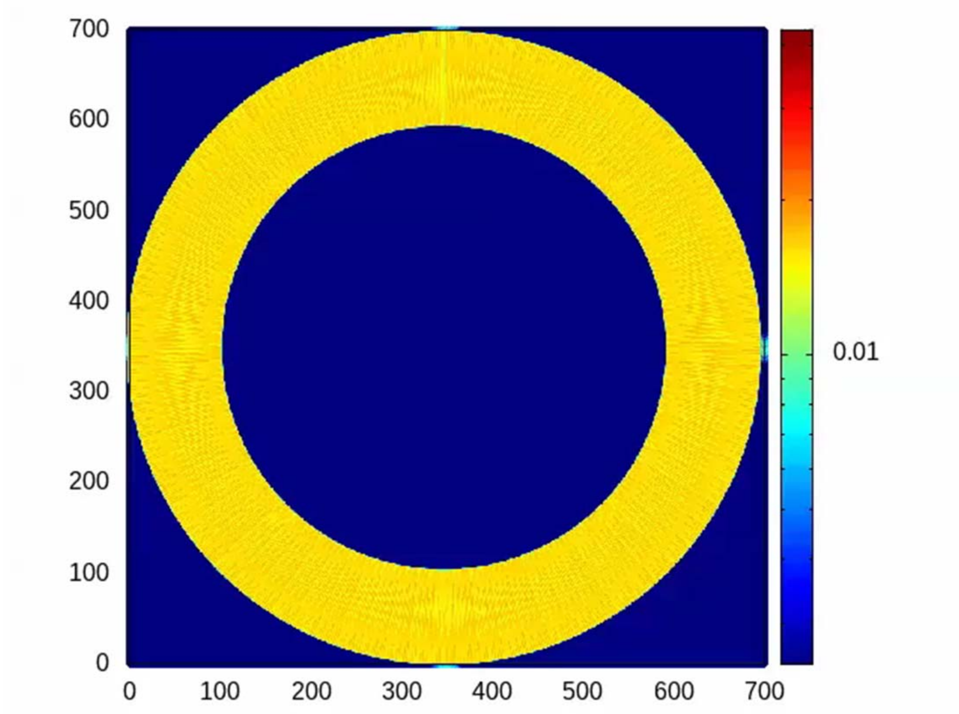
B(r) = B0 * r / Ri

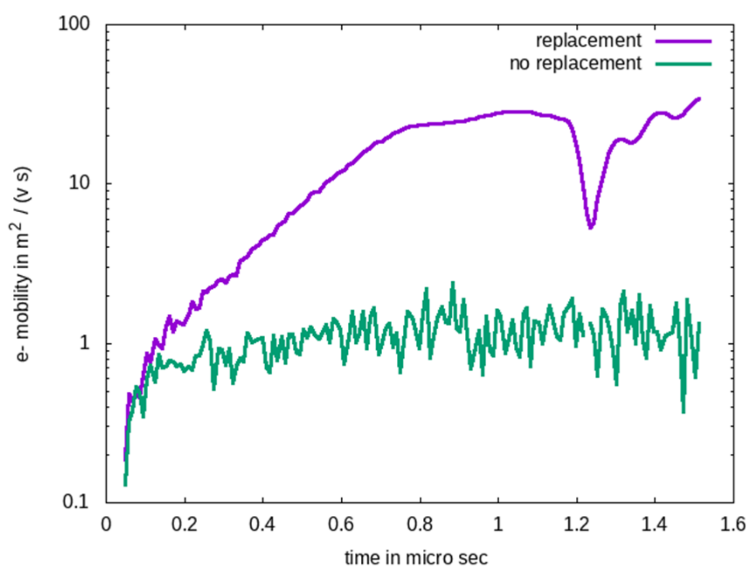
Ez = 2e4 V / m

Ti0 = Te0 = 0.0 K

Ion is Xe+

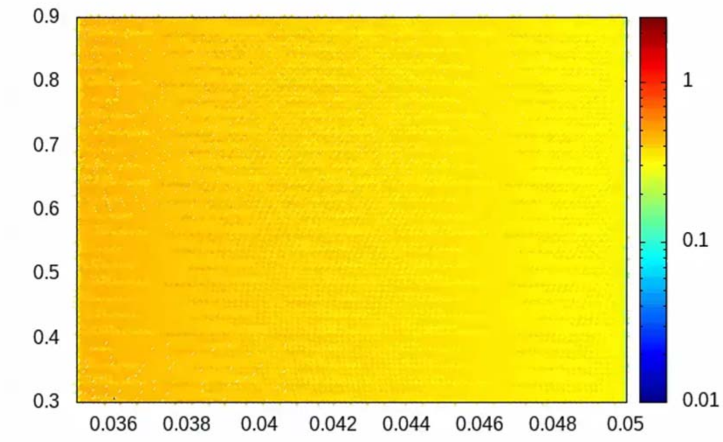
Full reflection walls , no sheath

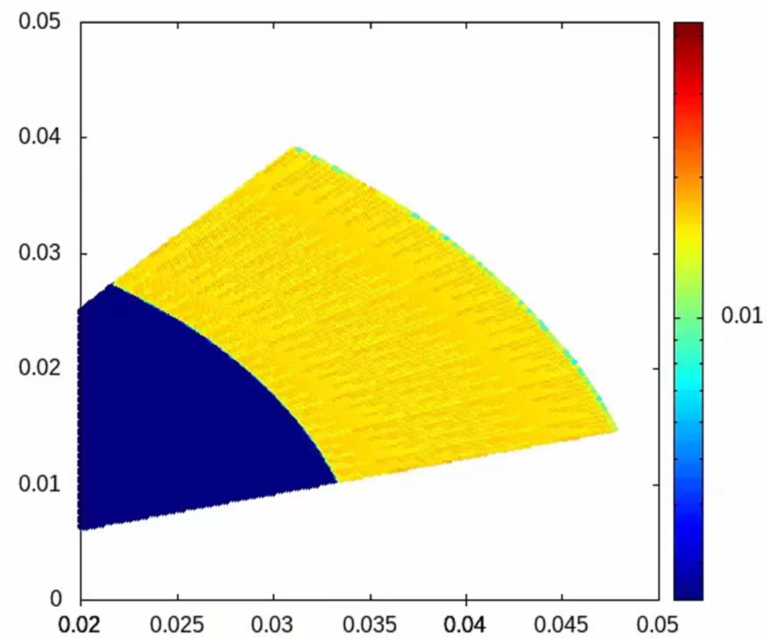




Particle replacement (fully inelastic collisions) substantially increase mobility

High Te temperature, ~ 90 eV
Radial mode number ~4, to be confirmed, similar to Taccogna?
Wavelength is increasing with time? Run longer? Currently 1.5 mks





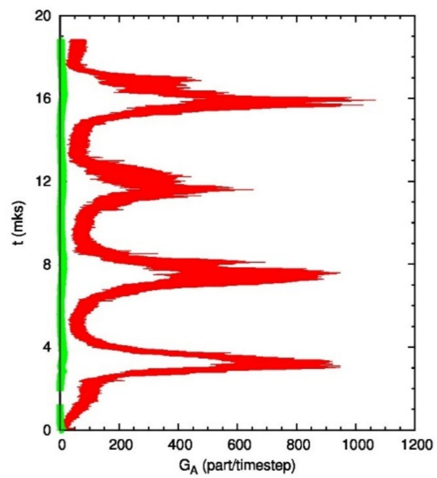
Theta-z simulations: Test case 2a
Comparison of narrow and wide box simulations
D. Sydorenko (2DPIC)

- Simulation 1: system size along the x-direction is 10 mm, along the y-direction is 25 mm, cathode end is metal, electron injection at y=24mm, injection flux equals to the electron flux minus ion flux at anode, x-averaged potential at the injection plane is -200 V, 1000 particles per cell.
- Simulation 2: same as simulation 1 but with the system size along the x-direction 80 mm,

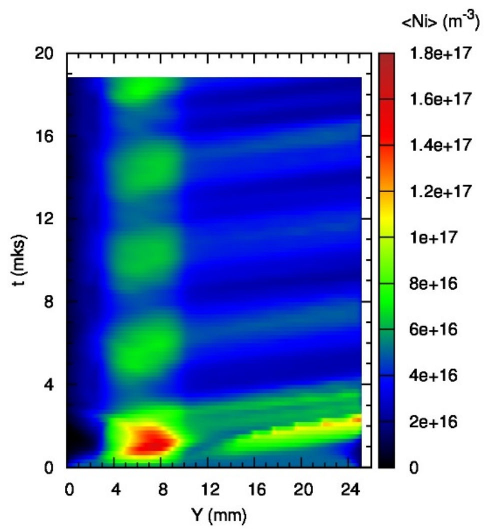
Both are with low current 100 A/m^2

Simulation 1

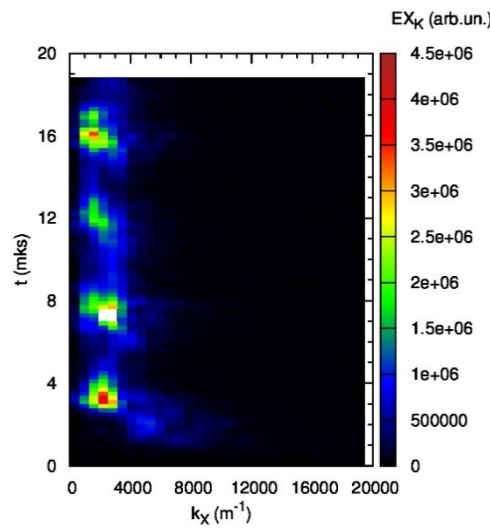
Anode fluxes of electrons (red) and ions (green) vs time



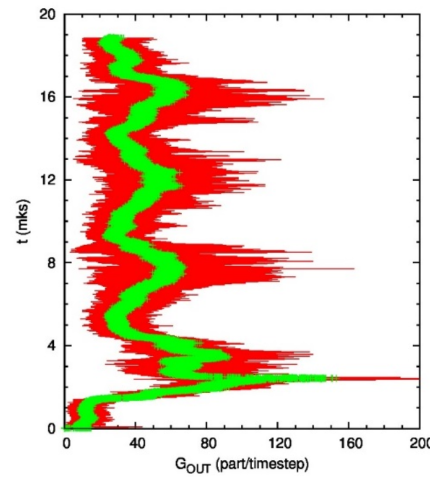
Ion density averaged along the X-direction vs time



Spectrum of EX over kx vs time

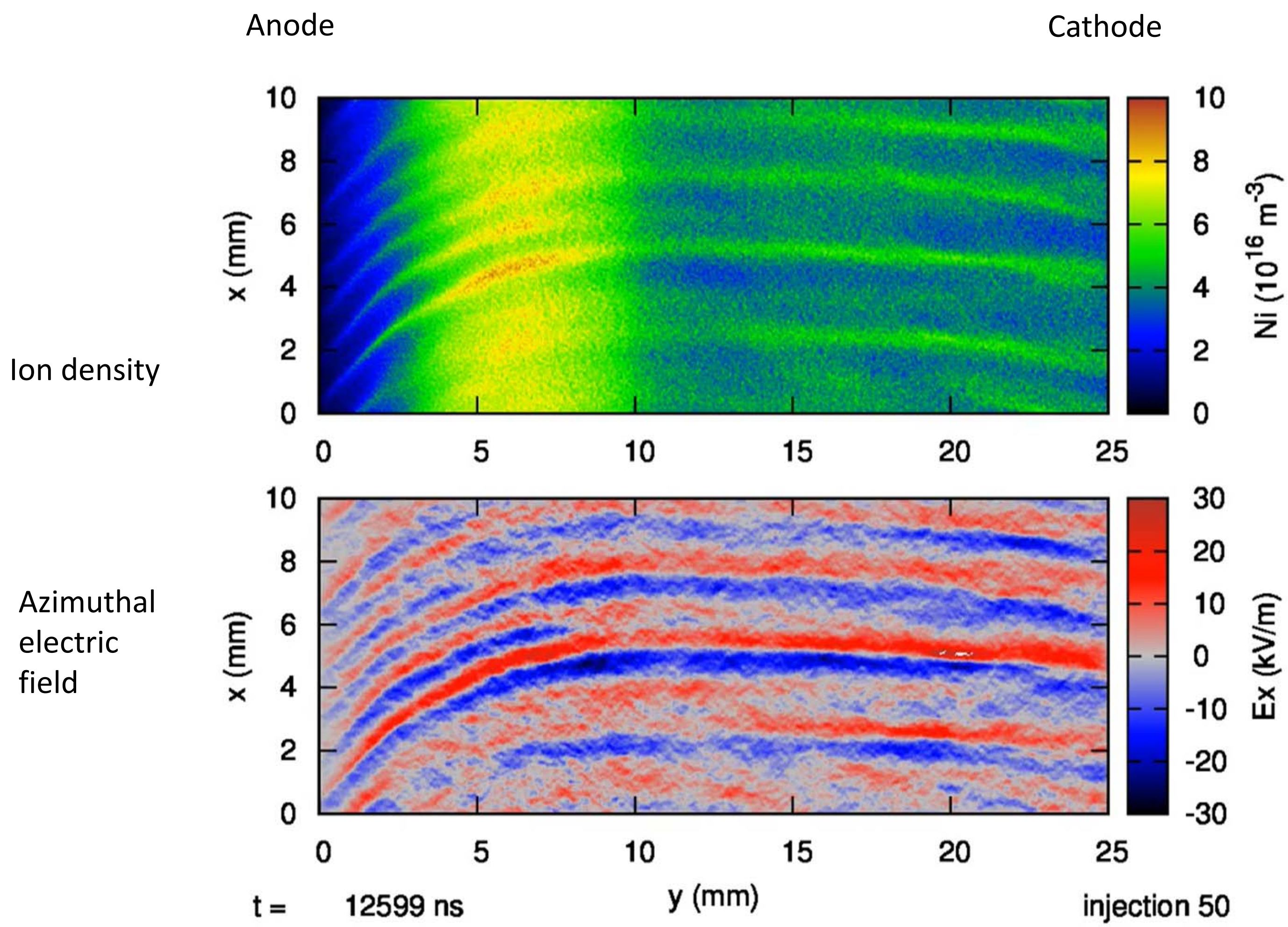


Fluxes of electrons (red) and ions (green) hitting the outflow boundary vs time



Large scale current oscillations, 250 KHz
similar to Boeuf, Garrigues 2018?
Coherent mode at $\lambda \simeq 2\ \mathrm{mm}$

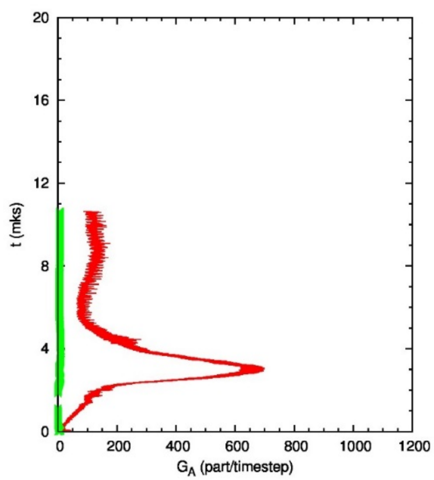
Traveling waves



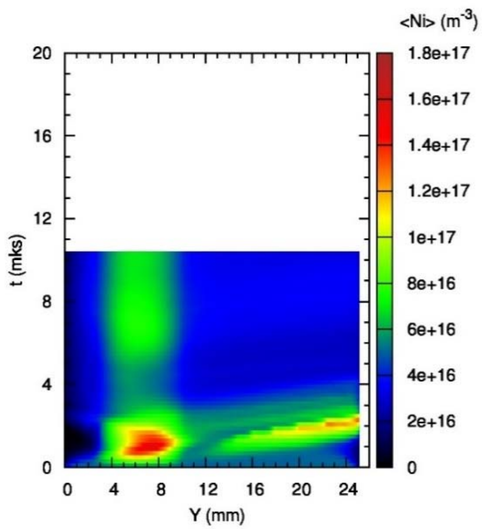
Simulation 2

Here the wall fluxes (given in particles colliding with the wall per time step) are divided by 8

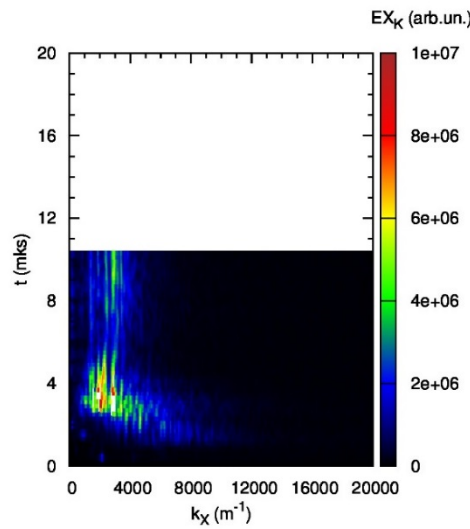
Anode fluxes of electrons (red) and ions (green) vs time



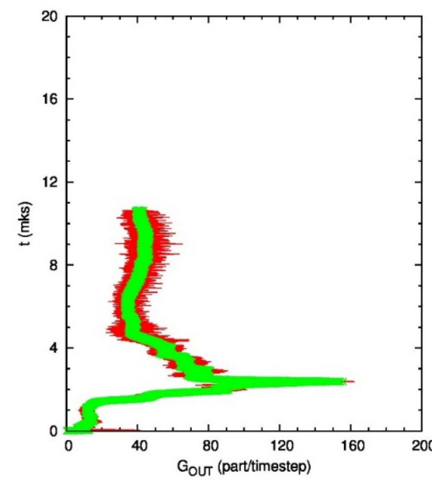
Ion density averaged along the X-direction vs time



Spectrum of EX over kx vs time



Fluxes of electrons (red) and ions (green) hitting the outflow boundary vs time

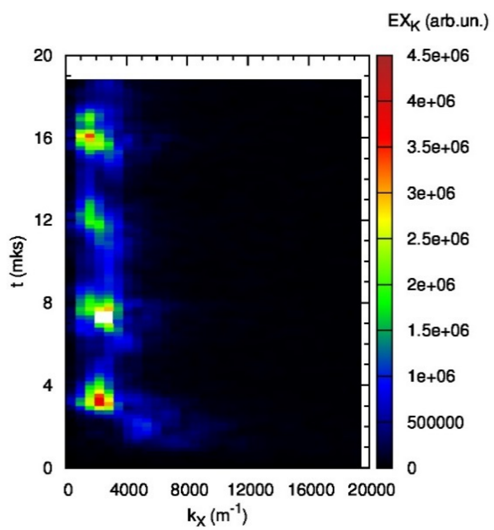


Large scale current oscillations seems disappear in a wide box

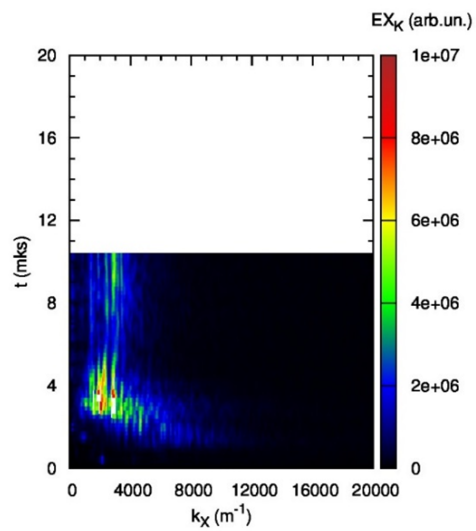
Short box

Wide box

Spectrum of EX
over kx vs time

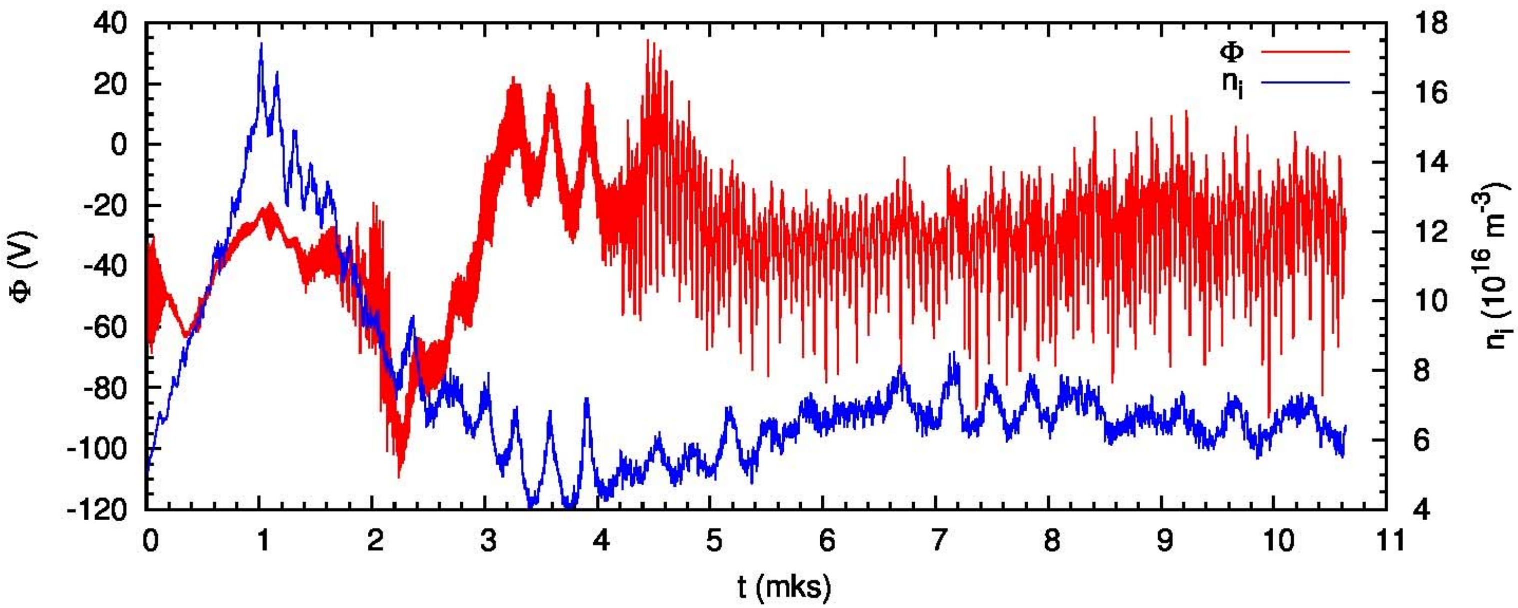


Spectrum of EX
over kx vs time

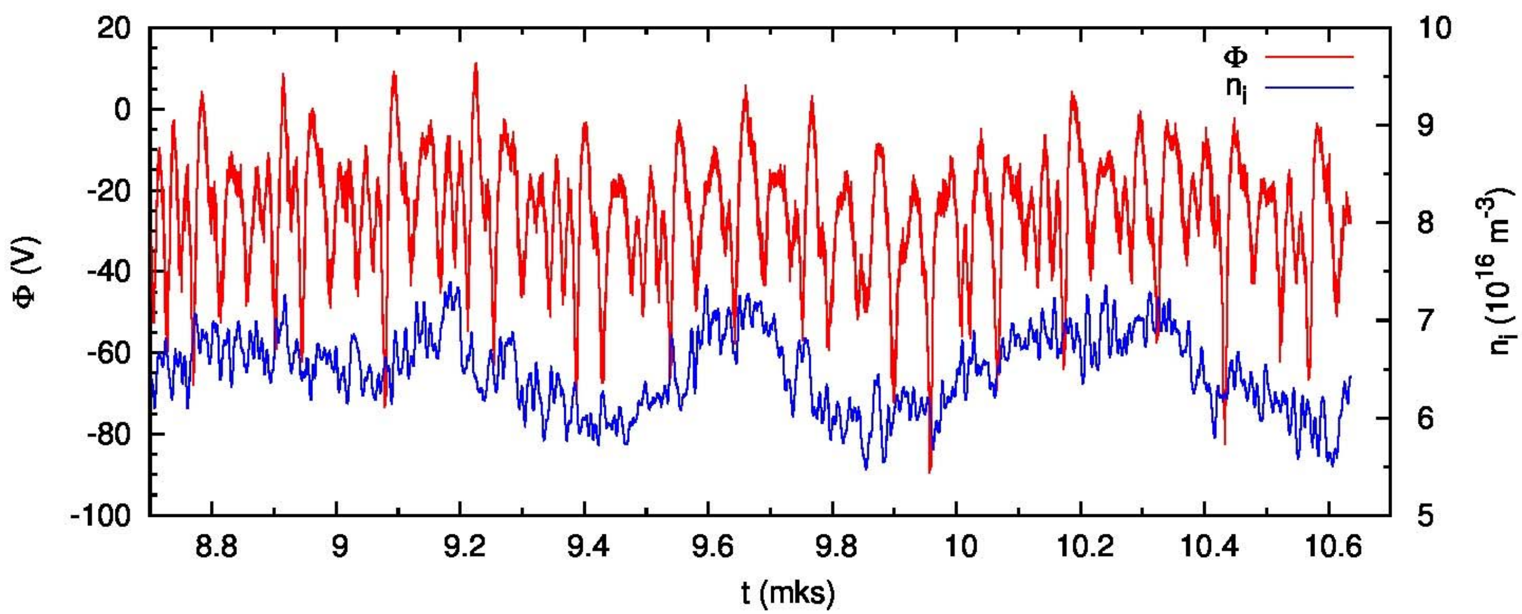


Large scale current oscillations seems disappear in a wide box

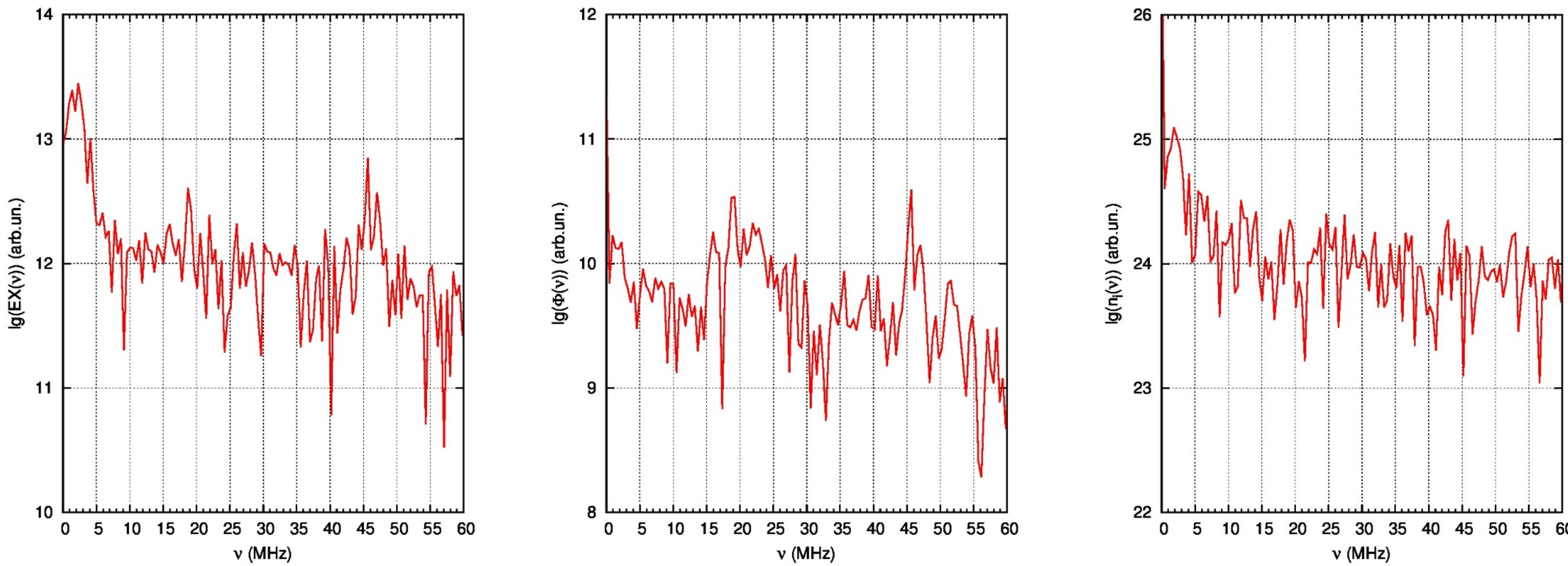
After t=4 mks high-frequency oscillations appear in the potential



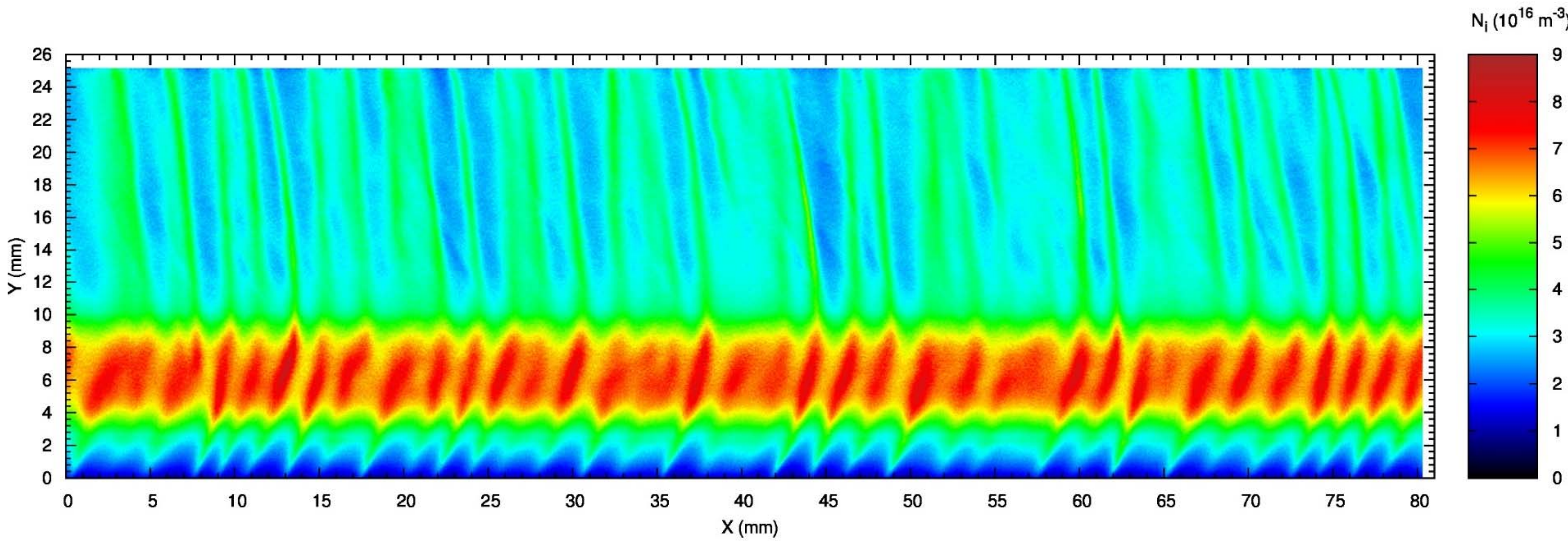
These oscillations have period of a few tens of ns (the period of the ECDI mode is hundreds of ns)



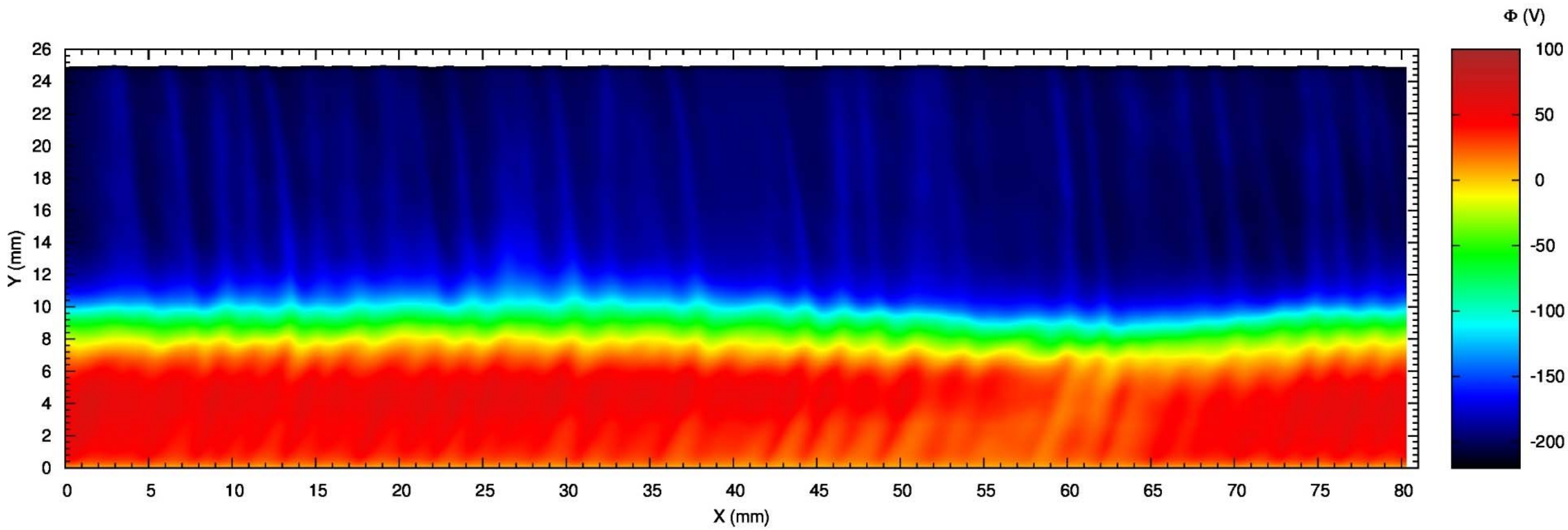
Frequency spectra of electrostatic field (left) and potential (middle) have frequencies of 20 and 45 MHz which are absent in the spectrum of the ion density (right).



Once again, nothing special in the snapshot of the ion density (10.6 mks)

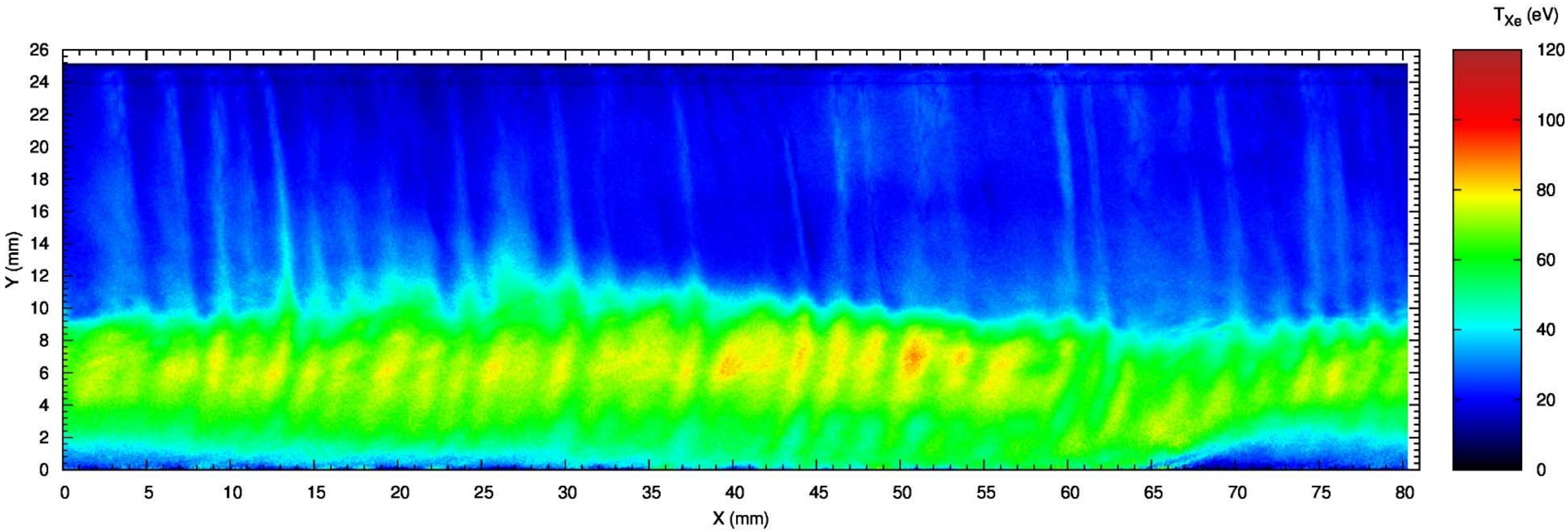


Meanwhile, the lines of equal potential are no longer parallel to the anode surface



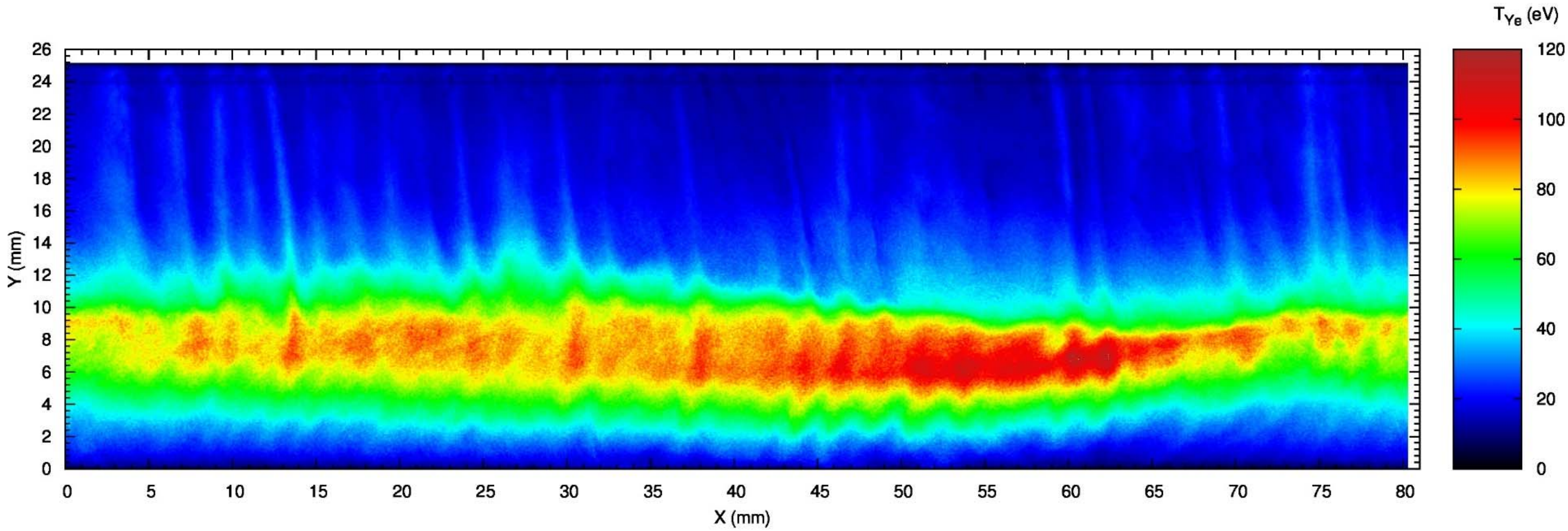
t = 10.6 mks

There is modulation in the electron temperature (for electron motion in the azimuthal direction)



t = 10.6 mks

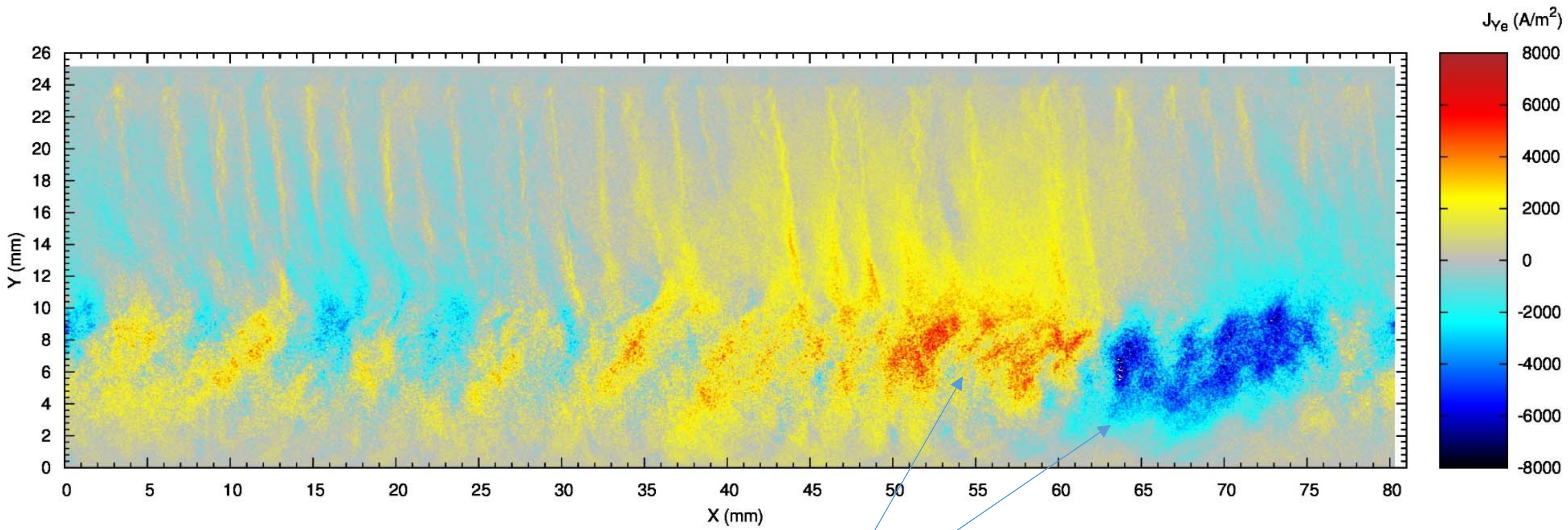
And for the electron temperature for motion along the axial direction



Note that the axial temperature is higher than the azimuthal one.

t = 10.6 mks

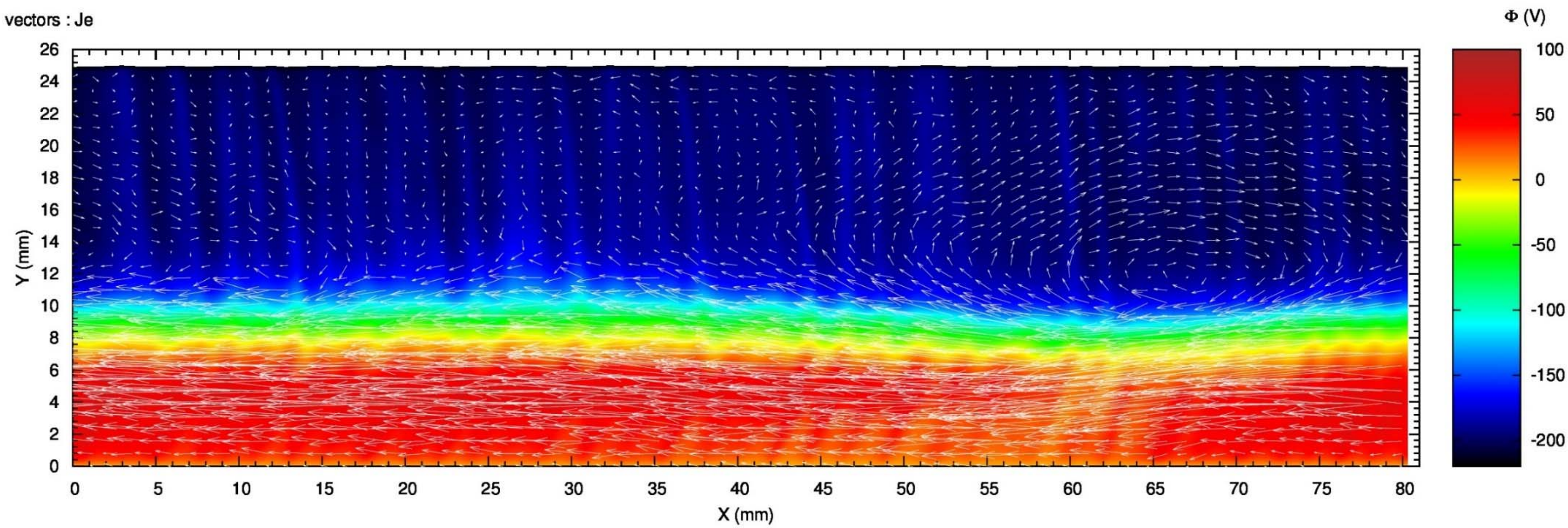
And for the axial electron current



t = 10.6 mks

Elecron current vortex

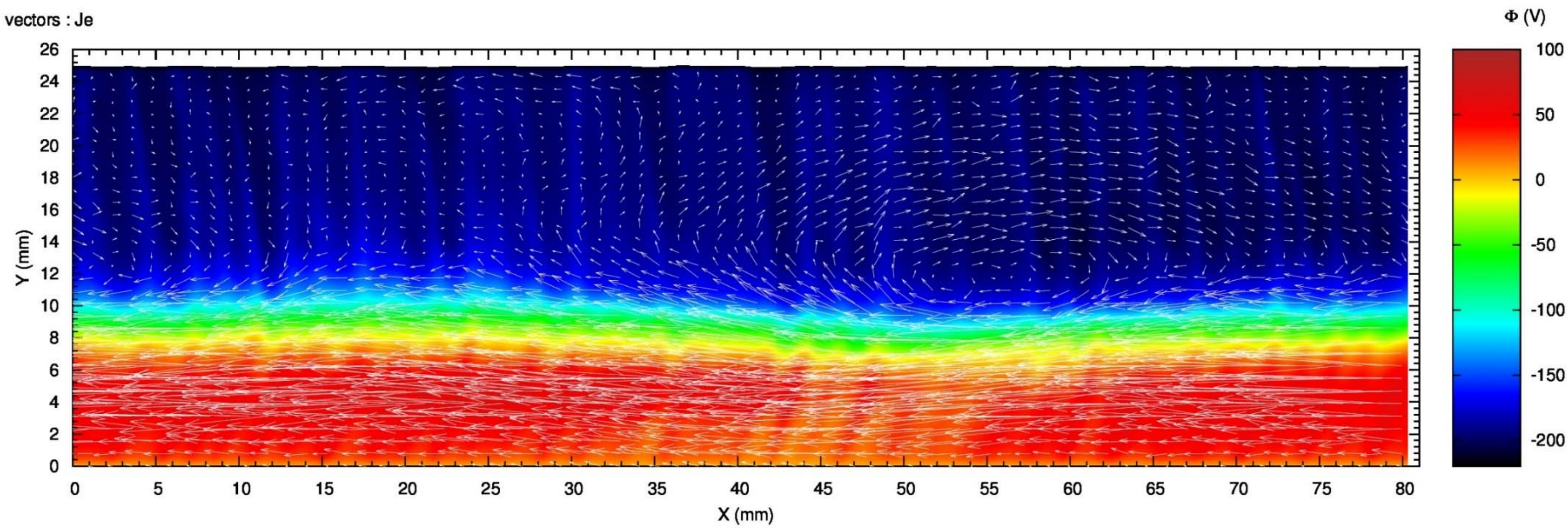
In fact, the electron current forms a vortex (or two) in the transport region (minimal axial potential gradient)



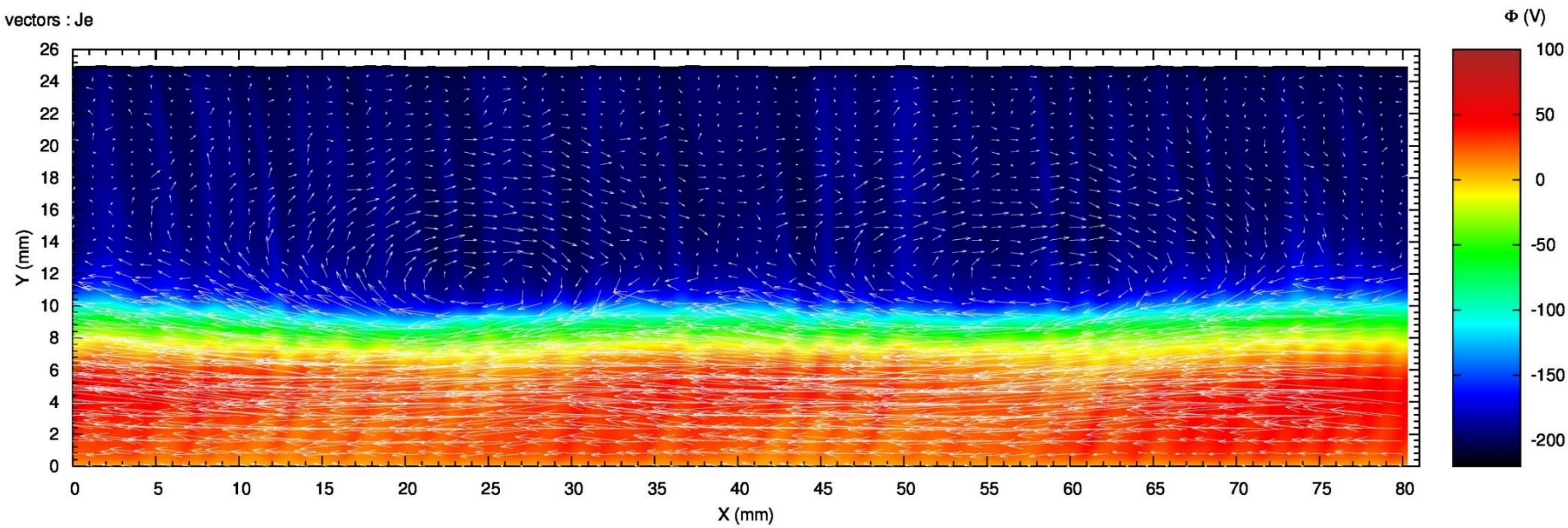
Theta –axial 2D simulations

- In the wide simulation, there are perturbations with short transverse wavelength (~2mm) and low frequency (~2.5 MHz). The speed of propagation of these perturbations in the azimuthal direction is ~5 km/s. Similar to Boeuf et al. It is present in both wide and narrow systems. The axial electron energy is larger than the azimuthal. Similar to Boeuf, LLP?
- In wide box, there is a higher frequency (20-45MHz) longer wavelength (~4cm) perturbation of electron parameters (temperatures and currents). The azimuthal velocity of propagation of these perturbations is ~(1-2)x1000 km/s, which is of the order of the ExB drift velocity. The perturbation also is accompanied by a strong vortex of the electron current. This mode is not found in the narrow system simulation.

Potential and electron currents at t=10.2 mks



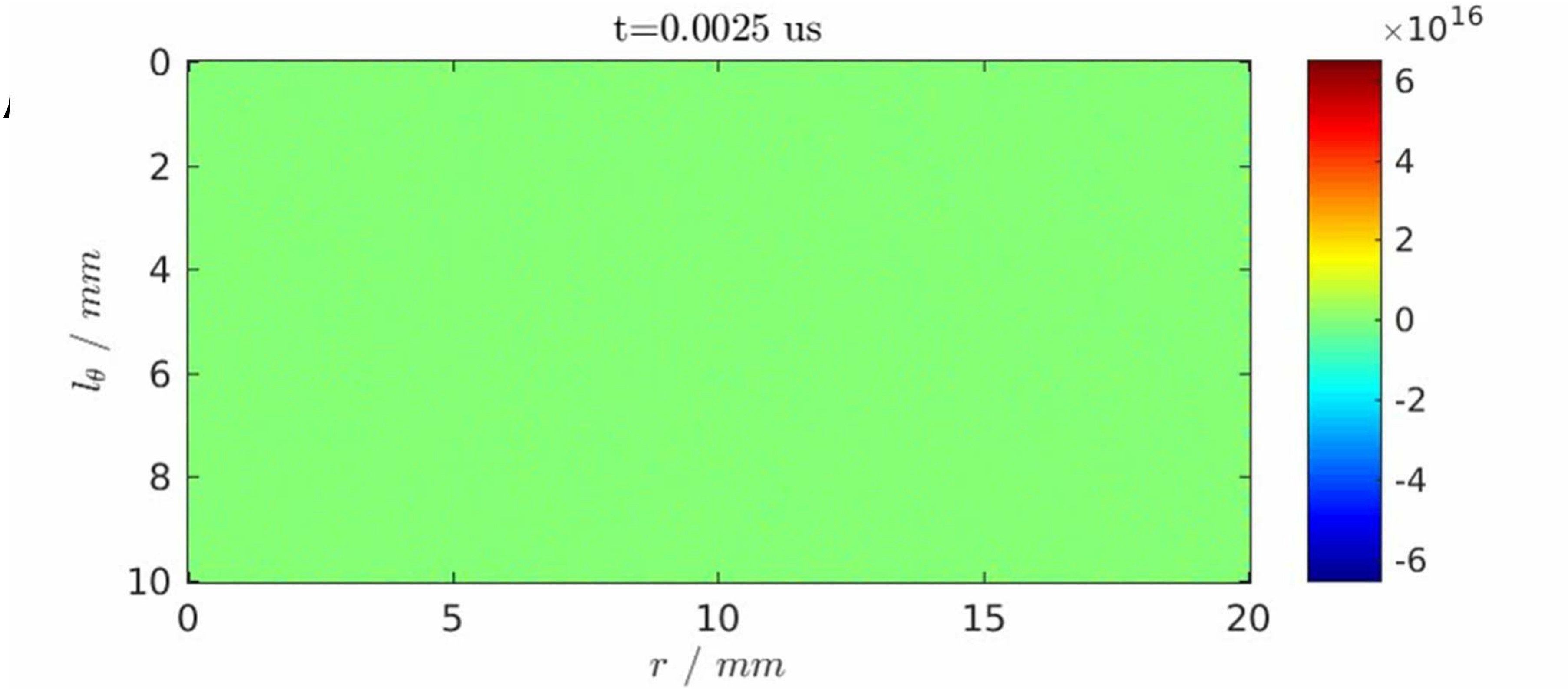
Potential and electron currents at t=10.4 mks



In Conclusions

- 1D nonlinear simulations are generally similar among several groups (Laplace, Bari, Usask)
  - Cnoidal waves in ion density, benign electron density, $k_{y0}V_{E0} = \omega_{ce}$ cyclotron resonance modes, condensation toward longer wavelengths, the mode frequency around ω_{pi} , axial current depends on the reinjection parameters/length, increases transport, ion trapping signatures, somewhat sensitive to simulation parameters (NPP), puzzling dependence of anomalous current (linear with E, but strong dependence on the magnetic field? Further studies are warranted but it is not clear if all relevant physics is included in such 1D.
- 2D (theta-radial) in many aspects (cnoidal waves, ω_{pi} harmonics) are similar to 1D, but radial modes included-> slow but intense MTSI modes, parallel heating, and structures in axial current. What determines the radial mode numbers? Different boundary conditions? Comparison is needed: Usask, Bari, Texas-AM (Hara)
- 2D theta-z simulations show common features: 2mm wave, higher axial temperature (Laplace, LLP, Usask). Wide box simulations (Usask) show high frequency modes, electron streamers in elecron current, and long wavelength modes/vortex. High frequency modes in HT (Litvak et al, Lazurenko et al)?
- From the theory perspective, the cyclotron resonances are not sufficiently broadened in 1D simulations, nor in 2D (theta-radial). It is somewhat unexpected that finite kz modes (along the magnetic field) do not make the mode ion-sound like via the linear mechanism (kz is not large enough, sheath effect) . Stochastic regimes in 2D theta-z and quasilinear regimes applicability are not clear

Sheath boundary conditions also give similar result: The inner part of plasma is insulated from boundaries and have much smaller effective k



Lower-hybrid waves

Plasma density and electric field fluctuations: cold plasma

Ion inertia

$$m_i \frac{\partial \tilde{\mathbf{V}}_i}{\partial t} = e\mathbf{E} = -e\nabla\phi$$

Restoring force is provided by the compressibility of the electron motion across the magnetic (electron inertia)

$$m_e \frac{\partial \tilde{\mathbf{V}}_e}{\partial t} = -e\left(\mathbf{E} + \mathbf{V_e} \times \mathbf{B}\right)$$

$$\tilde{\mathbf{V}}_\mathbf{e} = \frac{\tilde{\mathbf{E}} \times \mathbf{B}}{B^2} - \frac{1}{\omega_{ce} B} \frac{\partial}{\partial t} \tilde{\mathbf{E}}_\perp$$

$$\nabla \cdot \tilde{\mathbf{V}}_\mathbf{e} = -\frac{1}{\omega_{ce} B} \frac{\partial}{\partial t} \nabla \cdot \tilde{\mathbf{E}}_\perp \neq 0$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot \left(n \tilde{\mathbf{V}}_i\right) = 0 \quad \longleftrightarrow \quad \frac{\partial \tilde{n}}{\partial t} + \nabla \cdot \left(n \tilde{\mathbf{V}}_e\right) = 0$$

Balance of ion inertia against the electron transverse inertia: lower-hybrid wave →

$$\omega^2 = \omega_{ce}\omega_{ci}$$